## Calculation of the Surface Area by Geographic Coordinates Exercise 02

The Global Position (NAVSTAR) System that was discussed in Lesson 1 can be used for surveying the location and areas of pastures, forests, lakes, and rangelands. The estimate of surface areas is one of the more useful applications of this technology.

Determination of area from coordinates is a simple process for a closed polygon with known coordinates for each corner. The area is equal to $1 / 2$ the sum of the products obtained by multiplying each $Y$-coordinate by the difference between the adjacent $X$ coordinates. The $X$-coordinates must always be taken in a sequential order around the polygon we typically go clockwise. This rule can also be stated as follows: The area is equal to $1 / 2$ the sum of the products obtained by multiplying each $X$-coordinate by the difference between the adjacent $Y$-coordinates, taken in the same order around the figure.

For the pasture below, the rule is applied in the following relation:

$$
\text { Area }=1 / 2\left[X_{A}\left(Y_{\mathrm{E}}-Y_{\mathrm{B}}\right)+X_{\mathrm{B}}\left(Y_{\mathrm{A}}-Y_{\mathrm{C}}\right)+X_{\mathrm{C}}\left(Y_{\mathrm{B}}-Y_{\mathrm{D}}\right)+X_{\mathrm{D}}\left(Y_{\mathrm{C}}-Y_{\mathrm{E}}\right)+X_{\mathrm{E}}\left(Y_{\mathrm{D}}-Y_{\mathrm{A}}\right)\right]
$$

This formula is based on the summation of the areas of a series of trapezoids and can be derived from coordinate geometry.


## Solution

Calculation of Area from Coordinate Pairs

| Point | $X$ | $Y$ | Double Area |  |
| :---: | :---: | :---: | :---: | :--- |
| A | 0 | 0 | 0 | $=X_{A}\left(Y_{\mathrm{E}}-Y_{\mathrm{B}}\right)$ |
| B | 80 | 225 | $-23,200$ | $=X_{\mathrm{B}}\left(Y_{\mathrm{A}}-Y_{\mathrm{C}}\right)$ |
| C | 350 | 290 | 113,050 | $=X_{\mathrm{C}}\left(Y_{\mathrm{B}}-Y_{\mathrm{D}}\right)$ |
| D | 280 | -98 | 78,400 | $=X_{\mathrm{D}}\left(Y_{\mathrm{C}}-Y_{\mathrm{E}}\right)$ |
| E | 192 | 10 | $-18,816$ | $=X_{\mathrm{E}}\left(Y_{\mathrm{D}}-Y_{\mathrm{A}}\right)$ |
| A | 0 | 0 |  |  |
|  |  |  | 149434 | $=$ Double Area $\left(\mathrm{m}^{2}\right)$ |
| Area $=$ | 74,717 | $\mathrm{~m}^{2}$ | 7.4717 ha |  |

This problem can either be done by hand or you can build a spreadsheet to solve it. We suggest building a spreadsheet template that can be modified depending on the number of locations that are taken during the circuit. A template is a spreadsheet form or model that can be easily modified to solve similar problems that consist of the same types of data.

Of course, if you import the positions into a Geographic Information System (GIS) software package, the GIS will automatically calculate the perimeter and surface area of the polygon even if the positions are in the geographic (latitude/longitude) projection.

## Problems

1. You have obtained the following pairs of coordinates by GPS as you traversed around a location near Nimbol, India. You are using UTM coordinate system Zone 43 ( $72^{\circ} \mathrm{E}$ to $78^{\circ} \mathrm{E}$, Northern Hemisphere) with a WGS84 datum and all measurements are in meters. What is the area of this Field?

| Point | Easting | Northing | Elevation |
| :---: | :---: | :---: | :---: |
| A | 384420.7 | 2908829 | 278 |
| B | 384611.6 | 2908817 | 275 |
| C | 384621.6 | 2908759 | 276 |
| D | 384621.6 | 2908705 | 279 |
| E | 384418.9 | 2908723 | 280 |
| F | 384418.9 | 2908723 | 280 |
| A | 384420.7 | 2908829 | 278 |

2. You have obtained the following pairs of coordinates by GPS as you traversed around a pasture on In India. You are using UTM coordinate system Zone 43 ( $72^{\circ}$ E to $78^{\circ}$ E, Northern Hemisphere) with a WGS1984 datum and all measurements are in meters. What is the area of this pasture?

| Point | Easting | Northing | Elevation |
| :--- | :--- | :--- | :--- |
| A | 384605 | 2908853 | 274 |
| B | 384750 | 2908881 | 279 |
| C | 384773 | 2908877 | 279 |
| D | 384830 | 2908883 | 279 |
| E | 384845 | 2908766 | 278 |
| F | 384853 | 2908759 | 278 |
| G | 384924 | 2908772 | 280 |
| H | 384990 | 2908775 | 281 |
| I | 385025 | 2908781 | 281 |
| J | 385042 | 2908710 | 281 |
| K | 384970 | 2908685 | 281 |
| L | 384982 | 2908638 | 280 |
| M | 384910 | 2908611 | 280 |
| N | 384904 | 2908644 | 280 |
| O | 384830 | 2908632 | 282 |
| P | 384757 | 2908595 | 286 |
| Q | 384735 | 2908585 | 286 |
| R | 384679 | 2908569 | 285 |
| S | 384646 | 2908565 | 284 |
| T | 384624 | 2908706 | 279 |

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| $A$ | 384605 | 2908853 | 274 |
| :--- | :--- | :--- | :--- |

3. You have obtained the following pairs of coordinates by GPS as you traversed around a pasture near Nimbol, India. You are using UTM coordinate system Zone 43 ( $72^{\circ} \mathrm{E}$ to $78^{\circ} \mathrm{E}$, Northern Hemisphere) with a WGS1984 datum and all measurements are in meters. What is the area of this pasture?

| Point | Easting | Northing | Elevation |
| :--- | :--- | :--- | :--- |
| A | 384270.9 | 2907827 | 274.271 |
| B | 384279.2 | 2907902 | 274.792 |
| C | 384296 | 2907979 | 274.703 |
| D | 384342.8 | 2907981 | 272.392 |
| E | 384377.9 | 2907988 | 269.909 |
| F | 384406.4 | 2907911 | 268.509 |
| G | 384408 | 2907877 | 268.414 |
| H | 384421.4 | 2907850 | 267.953 |
| I | 384439.8 | 2907825 | 268.305 |
| J | 384458.2 | 2907802 | 269.327 |
| K | 384521.8 | 2907752 | 273.569 |
| L | 384495 | 2907628 | 277.773 |
| M | 384424.8 | 2907640 | 274.982 |
| N | 384424.8 | 2907640 | 274.982 |
| A | 384270.9 | 2907827 | 274.271 |

